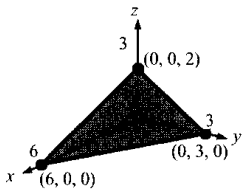
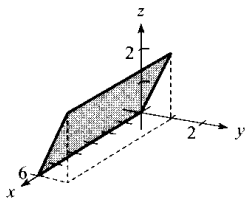
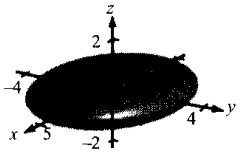
47.



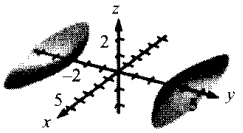
49.



51.



53.



$$\frac{x^2}{16} + \frac{y^2}{9} + z^2 = 1$$

55. *a*) $x^2 + y^2 - 2z + 2 = 0$

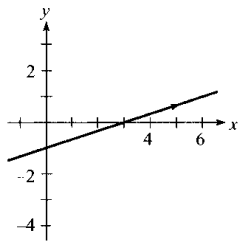
b) $4\pi \approx 12{,}6 \text{ cm}^3$

c) $\dfrac{225\pi}{64} \approx 11{,}0 \text{ cm}^3$

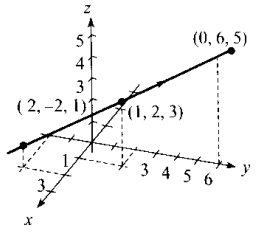
57. *a*) $\left(4, \dfrac{3\pi}{4}, 2\right)$ *b*) $\left(2\sqrt{5}, \dfrac{3\pi}{4}, \arccos\left[\dfrac{\sqrt{5}}{5}\right]\right)$

59. *a*) $r^2 \cos 2\theta = 2z$ *b*) $\rho = 2 \sec 2\theta \cos \phi \operatorname{cosec}^2 \phi$

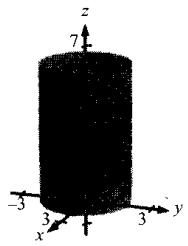
23.



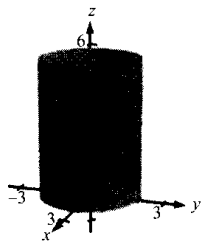
25.



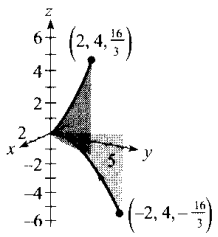
27.



29.



31.



CAPÍTULO 11

Sección 11.1 *(página 1052)*

1. $(-\infty, 0) \cup (0, \infty)$ **3.** $(0, \infty)$

5. $[0, \infty)$ **7.** $(-\infty, \infty)$

9. *a*) $\dfrac{1}{2}\mathbf{i}$ *b*) $\mathbf{j}$

c) $\dfrac{1}{2}(s + 1)^2\mathbf{i} - s\mathbf{j}$ *d*) $\dfrac{1}{2}\Delta t(\Delta t + 4)\mathbf{i} - \Delta t\mathbf{j}$

11. *a*) $\ln 2\mathbf{i} + \dfrac{1}{2}\mathbf{j} + 6\mathbf{k}$ *b*) No es posible

c) $\ln (t - 4)\mathbf{i} + \dfrac{1}{t - 4}\mathbf{j} + 3(t - 4)\mathbf{k}$

d) $\ln (1 + \Delta t)\mathbf{i} - \dfrac{\Delta t}{1 + \Delta t}\mathbf{j} + 3\Delta t\mathbf{k}$

13. $\sqrt{1 + t^2}$

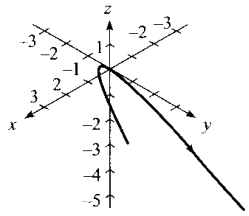
15. $t^2(5t - 1)$ El producto escalar es un número (un escalar)

17. b **18.** c **19.** d **20.** a

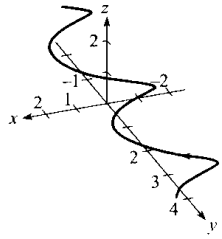
21. *a*) (−20, 0, 0) *b*) (10, 20, 10)

c) (0, 0, 20) *d*) (20, 0, 0)

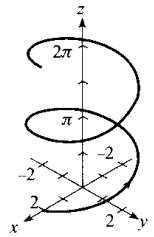
33. Parábola



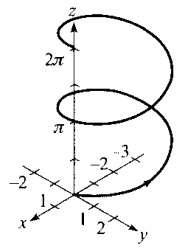
35. Hélice



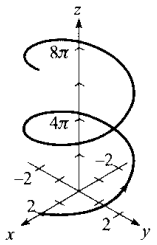
37.



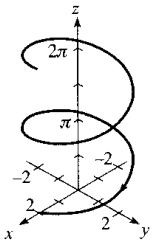
a) La hélice se traslada dos unidades hacia atrás por el eje x



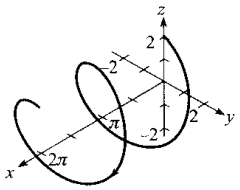
b) La altura de la hélice crece a ritmo más rápido



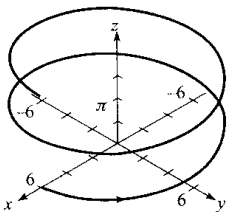
c) La orientación de la gráfica se invierte



d) El eje de la hélice es el eje x



e) El radio de la hélice crece de 2 a 6



39. $\mathbf{r}(t) = t\mathbf{i} + (4 - t)\mathbf{j}$

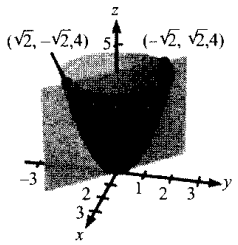
41. $\mathbf{r}(t) = 5 \cos t\mathbf{i} + 5 \operatorname{sen} t\mathbf{j}$

43. $\mathbf{r}(t) = \langle 2 - 2t, 3 + 5t, 8t \rangle$

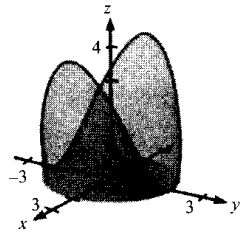
45. $\mathbf{r}_1(t) = t\mathbf{i}, \ 0 \leqslant t \leqslant 4$
$\mathbf{r}_2(t) = (4 - 4t)\mathbf{i} + 6t\mathbf{j}, \ 0 \leqslant t \leqslant 1$
$\mathbf{r}_3(t) = (6 - t)\mathbf{j}, \ 0 \leqslant t \leqslant 6$

47. $\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j}, \ 0 \leqslant t \leqslant 2$
$\mathbf{r}_2(t) = (2 - t)\mathbf{i}, \ 0 \leqslant t \leqslant 2$
$\mathbf{r}_3(t) = (4 - t)\mathbf{j}, \ 0 \leqslant t \leqslant 4$

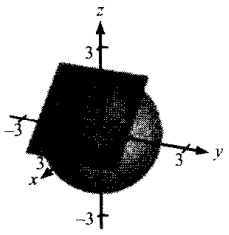
49. $\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + 2t^2\mathbf{k}$



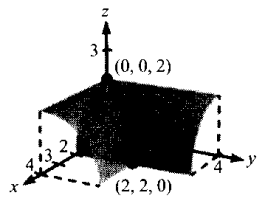
51. $\mathbf{r}(t) = 2 \operatorname{sen} t\mathbf{i} + 2 \cos t\mathbf{j} + 4 \operatorname{sen}^2 t\mathbf{k}$



53. $\mathbf{r}(t) = (1 + \operatorname{sen} t)\mathbf{i} + \sqrt{2} \cos t\mathbf{j} + (1 - \operatorname{sen} t)\mathbf{k}$ y
$\mathbf{r}(t) = (1 + \operatorname{sen} t)\mathbf{i} - \sqrt{2} \cos t\mathbf{j} + (1 - \operatorname{sen} t)\mathbf{k}$



55. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sqrt{4 - t^2}\mathbf{k}$



57. $2\mathbf{i} + 2\mathbf{j} + \frac{1}{2}\mathbf{k}$ **59.** **0**

61. No existe el límite **63.** $(-\infty, 0), (0, \infty)$

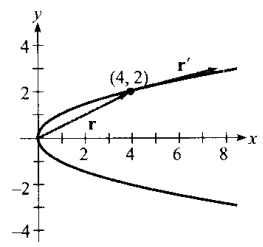
65. $[-1, 1]$ **67.** $\left(-\frac{\pi}{2} + n\pi, \frac{\pi}{2} + n\pi\right)$ **73.** Verdadero

74. Falso: $x = y = z = t^3$ representa una recta

Sección 11.2 *(página 1062)*

1. $\mathbf{r}(2) = 4\mathbf{i} + 2\mathbf{j}$

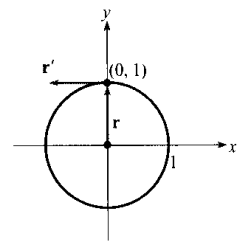
$\mathbf{r}'(2) = 4\mathbf{i} + \mathbf{j}$



$r'(t_0)$ es tangente a la curva en t_0

3. $\mathbf{r}\left(\frac{\pi}{2}\right) = \mathbf{j}$

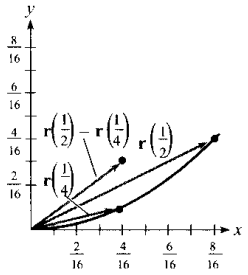
$\mathbf{r}'\left(\frac{\pi}{2}\right) = -\mathbf{i}$



$r'(t_0)$ es tangente a la curva en t_0

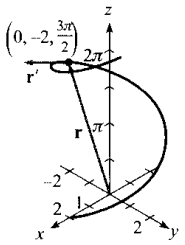
5. *a)* y *b)*

c) El vector $\dfrac{\mathbf{r}(\frac{1}{2}) - \mathbf{r}(\frac{1}{4})}{\frac{1}{2} - \frac{1}{4}}$ aproxima al vector tangente $\mathbf{r}'(\frac{1}{4})$



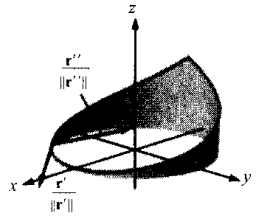
7. $\mathbf{r}\left(\dfrac{3\pi}{2}\right) = -2\mathbf{j} + \left(\dfrac{3\pi}{2}\right)\mathbf{k}$

$\mathbf{r}'\left(\dfrac{3\pi}{2}\right) = 2\mathbf{i} + \mathbf{k}$



9. $\dfrac{\mathbf{r}'(-\frac{1}{4})}{\|\mathbf{r}'(-\frac{1}{4})\|} = \dfrac{1}{\sqrt{4\pi^2 + 1}}(\sqrt{2}\pi\mathbf{i} + \sqrt{2}\pi\mathbf{j} - \mathbf{k})$

$\dfrac{\mathbf{r}''(-\frac{1}{4})}{\|\mathbf{r}''(-\frac{1}{4})\|} = \dfrac{1}{2\sqrt{\pi^4 + 4}}(-\sqrt{2}\pi^2\mathbf{i} + \sqrt{2}\pi^2\mathbf{j} + 4\mathbf{k})$



11. $6\mathbf{i} - 14t\mathbf{j} + 3t^2\mathbf{k}$

13. $-3a \operatorname{sen} t \cos^2 t\mathbf{i} + 3a \operatorname{sen}^2 t \cos t\mathbf{j}$

15. $-e^{-t}\mathbf{i}$ **17.** $\langle \operatorname{sen} t + t\cos t, \cos t - t \operatorname{sen} t, 1\rangle$

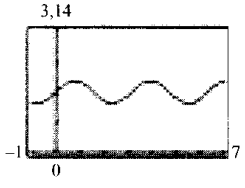
19. *a)* $\mathbf{i} + 3\mathbf{j} + 2t\mathbf{k}$ *b)* $2\mathbf{k}$ *c)* $8t + 9t^2 + 5t^4$

d) $-\mathbf{i} + (9 - 2t)\mathbf{j} + (6t - 3t^2)\mathbf{k}$

e) $8t^3\mathbf{i} + (12t^2 - 4t^3)\mathbf{j} + (3t^2 - 24t)\mathbf{k}$

f) $\dfrac{10 + 2t^2}{\sqrt{10 + t^2}}$

21. $\theta(t) = \arccos\left(\dfrac{-7 \operatorname{sen} t \cos t}{\sqrt{9 \operatorname{sen}^2 t + 16\cos^2 t}\,\sqrt{9\cos^2 t + 16 \operatorname{sen}^2 t}}\right)$



Máximo: $\theta\left(\dfrac{\pi}{4}\right) = \theta\left(\dfrac{5\pi}{4}\right) \approx 1{,}855$

Mínimo: $\theta\left(\dfrac{3\pi}{4}\right) = \theta\left(\dfrac{7\pi}{4}\right) \approx 1{,}287$

Ortogonales: $\theta\left(\dfrac{n\pi}{2}\right) = \left(\dfrac{\pi}{2}\right)$

23. $(-\infty, 0), (0, \infty)$ **25.** $\left(\dfrac{n\pi}{2}, \dfrac{(n+1)\pi}{2}\right)$

27. $(-\infty, \infty)$ **29.** $(-\infty, 0), (0, \infty)$

31. $\left(-\dfrac{\pi}{2} + n\pi, \dfrac{\pi}{2} + n\pi\right)$, n entero

33. $\mathbf{r}'(t) = 3\mathbf{i} - 2t\mathbf{j}$

35. Las tres componentes de $\mathbf{u}$ son funciones crecientes de t en $t = t_0$.

37. $t^2\mathbf{i} + t\mathbf{j} + t\mathbf{k} + \mathbf{C}$ **39.** $\ln t\mathbf{i} + t\mathbf{j} - \frac{2}{5}t^{5/2}\mathbf{k} + \mathbf{C}$

41. $(t^2 - t)\mathbf{i} + t^4\mathbf{j} + 2t^{3/2}\mathbf{k} + \mathbf{C}$ **43.** $\operatorname{tg} t\mathbf{i} + \operatorname{arctg} t\mathbf{j} + \mathbf{C}$

45. $2e^{2t}\mathbf{i} + 3(e^t - 1)\mathbf{j}$ **47.** $600\sqrt{3}t\mathbf{i} + (-16t^2 + 600t)\mathbf{j}$

49. $\left(\dfrac{2 - e^{-t^2}}{2}\right)\mathbf{i} + (e^{-t} - 2)\mathbf{j} + (t + 1)\mathbf{k}$

51. $4\mathbf{i} + \dfrac{1}{2}\mathbf{j} - \mathbf{k}$ **53.** $a\mathbf{i} + a\mathbf{j} + \dfrac{\pi}{2}\mathbf{k}$

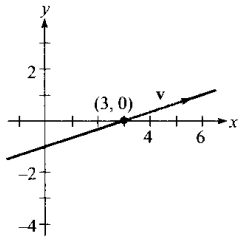
63. Falso: si tomamos $\mathbf{r}(t) = \cos t\mathbf{i} + \operatorname{sen} t\mathbf{j} + \mathbf{k}$, entonces $\dfrac{d}{dt}[\|\mathbf{r}(t)\|] = 0$, pero $\|\mathbf{r}'(t)\| = 1$

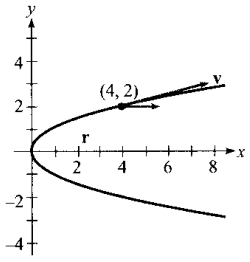
64. Falso: $D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}(t) \cdot \mathbf{u}'(t) + \mathbf{r}'(t) \cdot \mathbf{u}(t)$

Sección 11.3 *(página 1071)*

1. $\mathbf{v}(1) = 3\mathbf{i} + \mathbf{j}$
$\mathbf{a}(1) = \mathbf{0}$

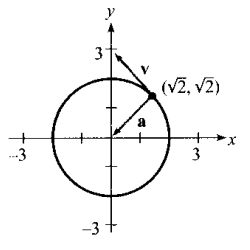
3. $\mathbf{v}(2) = 4\mathbf{i} + \mathbf{j}$
$\mathbf{a}(2) = 2\mathbf{i}$



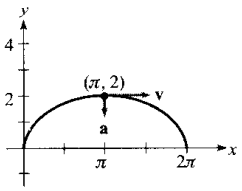


5. $\mathbf{v}\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$

$\mathbf{a}\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$



7. $\mathbf{v}(\pi) = 2\mathbf{i}$
$\mathbf{a}(\pi) = -\mathbf{j}$



9. $\mathbf{v}(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
$s(t) = \sqrt{14}$
$\mathbf{a}(t) = \mathbf{0}$

11. $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$
$s(t) = \sqrt{1 + 5t^2}$
$\mathbf{a}(t) = 2\mathbf{j} + \mathbf{k}$

13. $\mathbf{v}(t) = \mathbf{i} + \mathbf{j} - \dfrac{t}{\sqrt{9 - t^2}}\mathbf{k}$
$s(t) = \sqrt{\dfrac{18 - t^2}{9 - t^2}}$
$\mathbf{a}(t) = \dfrac{-9}{(9 - t^2)^{3/2}}\mathbf{k}$

15. $\mathbf{v}(t) = 4\mathbf{i} - 3 \operatorname{sen} t\mathbf{j} + 3 \cos t\mathbf{k}$
$s(t) = 5$
$\mathbf{a}(t) = -3 \cos t\mathbf{j} - 3 \operatorname{sen} t\mathbf{k}$

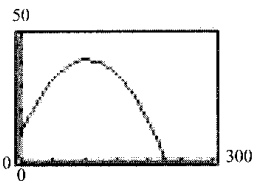
17. *a)* $x = 1 + t$
$y = -1 - 2t$
$z = \dfrac{1}{4} + \dfrac{3}{4}t$

b) (1,100, −1,200, 0,325)

19. $\mathbf{v}(t) = t(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{r}(t) = \dfrac{t^2}{2}(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{r}(2) = 2(\mathbf{i} + \mathbf{j} + \mathbf{k})$

21. $\mathbf{v}(t) = \left(\dfrac{t^2}{2} + \dfrac{9}{2}\right)\mathbf{j} + \left(\dfrac{t^2}{2} - \dfrac{1}{2}\right)\mathbf{k}$
$\mathbf{r}(t) = \left(\dfrac{t^3}{6} + \dfrac{9}{2}t - \dfrac{14}{3}\right)\mathbf{j} + \left(\dfrac{t^3}{6} - \dfrac{1}{2}t + \dfrac{1}{3}\right)\mathbf{k}$
$\mathbf{r}(2) = \dfrac{17}{3}\mathbf{j} + \dfrac{2}{3}\mathbf{k}$

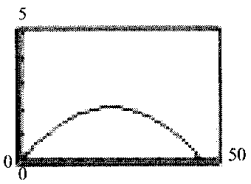
23. $\mathbf{r}(t) = 44\sqrt{3}t\mathbf{i} + (10 + 44t - 16t^2)\mathbf{j}$



25. $v_0 = 28{,}78$ pies/s, $\theta = 58{,}28°$

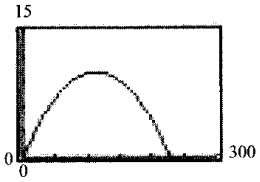
27. 1,91°

29. *a)*



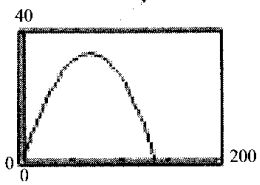
Altura máxima: 2,1 pies
Alcance: 46,6 pies

b)



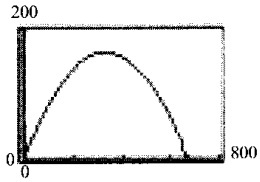
Altura máxima: 10,0 pies
Alcance: 227,8 pies

c)



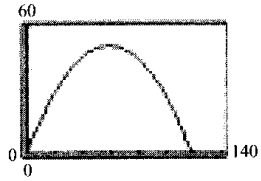
Altura máxima: 34,0 pies
Alcance: 136,1 pies

d)



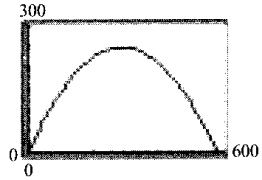
Altura máxima: 166,5 pies
Alcance: 666,1 pies

e)



Altura máxima: 51,0 pies
Alcance: 117,9 pies

f)



Altura máxima: 249,8 pies
Alcance: 576,9 pies

31. $\mathbf{r}(t) = (40\sqrt{2}t)\mathbf{i} + (40\sqrt{2}t - 16t^2)\mathbf{j}$
Dirección: $8\sqrt{2}(5\mathbf{i} + 2\mathbf{j})$
Velocidad: $8\sqrt{58} \approx 60{,}9$ pies/s

33. Altura máxima: 129,1 pies
Alcance: 886,3 pies

35. $\mathbf{v}(t) = b\omega[1 - \cos \omega t)\mathbf{i} + \operatorname{sen} \omega t\mathbf{j}]$
$\mathbf{a}(t) = b\omega^2(\operatorname{sen} \omega t\mathbf{i} + \cos \omega t\mathbf{j})$
a) $\|\mathbf{v}(t)\| = 0$ cuando $\omega t = 0, 2\pi, 4\pi, \ldots$
b) $\|\mathbf{v}(t)\|$ es máximo cuando $\omega t = \pi, 3\pi, \ldots$

37. $\mathbf{v}(t) = -b\omega \operatorname{sen} \omega t\mathbf{i} + \omega t \cos \omega t\mathbf{j}$
$\mathbf{v}(t) \cdot \mathbf{r}(t) = 0$

39. $\mathbf{a}(t) = -b\omega^2(\cos \omega t\mathbf{i} + \operatorname{sen} \omega t\mathbf{j}) = -\omega^2\mathbf{r}(t)$

41. $8\sqrt{10}$ pies/s

47. *a)* $\mathbf{v}(t) = -6 \operatorname{sen} t\mathbf{i} + 3 \cos t\mathbf{j}$

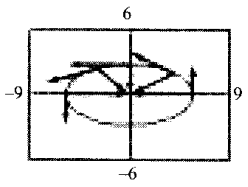
$\|\mathbf{v}(t)\| = 3\sqrt{3 \operatorname{sen}^2 t + 1}$

$\mathbf{a}(t) = -6 \cos t\mathbf{i} - 3 \operatorname{sen} t\mathbf{j}$

b)

t	0	$\pi/4$	$\pi/2$	$2\pi/3$	π
Velocidad	3	$3\sqrt{10}/2$	6	$3\sqrt{13}/2$	3

c)



d) La rapidez está creciendo cuando el ángulo entre **v** y **a** está en el intervalo $[0, \pi/2)$ y decreciendo cuando dicho ángulo está en el intervalo $(\pi/2, \pi]$.

Sección 11.4 *(página 1082)*

1. $\mathbf{T}(0) = \dfrac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$

$x = t$

$y = 0$

$z = t$

3. $\mathbf{T}(0) = \dfrac{\sqrt{5}}{5}(2\mathbf{j} + \mathbf{k})$

$x = 2$

$y = 2t$

$z = t$

5. $\mathbf{T}\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2}\langle -\sqrt{2}, \sqrt{2}, 0\rangle$

$x = \sqrt{2} - \sqrt{2}t$

$y = \sqrt{2} + \sqrt{2}t$

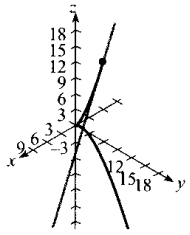
$z = 4$

7. $\mathbf{T}(3) = \frac{1}{19}\langle 1, 6, 18\rangle$

$x = 3 + t$

$y = 9 + 6t$

$z = 18 + 18t$



9. Recta tangente: $x = 1 + t$

$y = t$

$z = 1 + \dfrac{1}{2}t$

$\mathbf{r}(1,1) \approx \langle 1{,}1,\ 0{,}1,\ 1{,}05\rangle$

11. 1,2°

13. $\mathbf{v}(t) = 4\mathbf{i}$

$\mathbf{a}(t) = \mathbf{0}$

$\mathbf{T}(t) = \mathbf{i}$

$\mathbf{N}(t)$ no está definido. La trayectoria es una recta y la velocidad es constante.

15. $\mathbf{v}(t) = 8t\mathbf{i}$

$\mathbf{a}(t) = 8\mathbf{i}$

$\mathbf{T}(t) = \mathbf{i}$

$\mathbf{N}(t)$ no está definido. La trayectoria es una recta y la velocidad es variable.

17. $\mathbf{T} = \dfrac{\sqrt{2}}{2}(\mathbf{i} - \mathbf{j})$

$\mathbf{N} = \dfrac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{j})$

$a_{\mathbf{T}} = -\sqrt{2}$

$a_{\mathbf{N}} = \sqrt{2}$

19. $\mathbf{T} = \dfrac{\sqrt{2}}{2}(-\mathbf{i} + \mathbf{j})$

$\mathbf{N} = -\dfrac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{j})$

$a_{\mathbf{T}} = \sqrt{2}e^{\pi/2}$

$a_{\mathbf{N}} = \sqrt{2}e^{\pi/2}$

21. $\mathbf{T} = (\cos \omega t_0)\mathbf{i} + (\operatorname{sen} \omega t_0)\mathbf{j}$

$\mathbf{N} = (-\operatorname{sen} \omega t_0)\mathbf{i} + (\cos \omega t_0)\mathbf{j}$

$a_{\mathbf{T}} = \omega^2$

$a_{\mathbf{N}} = \omega^3 t_0$

23. $\mathbf{T}(t) = -\operatorname{sen}(\omega t)\mathbf{i} + \cos(\omega t)\mathbf{j}$

$\mathbf{N}(t) = -\cos(\omega t)\mathbf{i} - \operatorname{sen}(\omega t)\mathbf{j}$

$a_{\mathbf{T}} = 0$

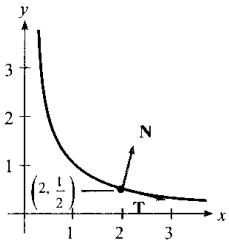
$a_{\mathbf{N}} = a\omega^2$

25. $\|\mathbf{v}(t)\| = a\omega$ La rapidez es constante, porque $a_{\mathbf{T}} = 0$.

27. $\mathbf{r}(2) = 2\mathbf{i} + \dfrac{1}{2}\mathbf{j}$

$\mathbf{T}(2) = \dfrac{\sqrt{17}}{17}(4\mathbf{i} - \mathbf{j})$

$\mathbf{N}(2) = \dfrac{\sqrt{17}}{17}(\mathbf{i} + 4\mathbf{j})$



29. *a)* $t = \dfrac{1}{2}$: $a_{\mathbf{T}} = \dfrac{\sqrt{2}\pi^2}{2}$, $a_{\mathbf{N}} = \dfrac{\sqrt{2}\pi^2}{2}$

$t = 1$: $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = \pi^2$

$t = \dfrac{3}{2}$: $a_{\mathbf{T}} = -\dfrac{\sqrt{2}\pi^2}{2}$, $a_{\mathbf{N}} = \dfrac{\sqrt{2}\pi^2}{2}$

b) $t = \dfrac{1}{2}$: Creciente

$t = 1$: Máximo

$t = \dfrac{3}{2}$: Decreciente

31. $\mathbf{T}(t) = \frac{\sqrt{14}}{14}(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$

$\mathbf{N}(t)$ no está definido

$a_{\mathbf{T}}$ no está definido

$a_{\mathbf{N}}$ no está definido

33. $\mathbf{T} = \frac{\sqrt{6}}{6}(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

$\mathbf{N} = \frac{\sqrt{30}}{30}(-5\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

$a_{\mathbf{T}} = \frac{5\sqrt{6}}{6}$

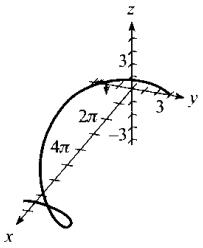
$a_{\mathbf{N}} = \frac{\sqrt{30}}{6}$

35. $\mathbf{T} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{j})$

$\mathbf{N} = -\mathbf{k}$

$a_{\mathbf{T}} = 0$

$a_{\mathbf{N}} = 3$



37. $\mathbf{T}\left(\frac{\pi}{2}\right) = \frac{\sqrt{17}}{17}(-4\mathbf{i} + \mathbf{k})$

$\mathbf{N}\left(\frac{\pi}{2}\right) = -\mathbf{j}$

$\mathbf{B}\left(\frac{\pi}{2}\right) = \frac{\sqrt{17}}{17}(\mathbf{i} + 4\mathbf{k})$

39. *a*) $4\sqrt{625\pi^2 + 1} \approx 314$ millas/h

b) $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = 1.000\pi^2$

41. $a_{\mathbf{T}} = \dfrac{-32(v_0 \operatorname{sen}\theta - 32t)}{\sqrt{v_0^2 \cos^2\theta + (v_0 \operatorname{sen}\theta - 32t)^2}}$

$a_{\mathbf{N}} = \dfrac{32v_0 \cos\theta}{\sqrt{v_0^2 \cos^2\theta + (v_0 \operatorname{sen}\theta - 32t)^2}}$

43. *a*) La componente centrípeta se cuadruplica.

b) La componente centrípeta queda dividida por dos.

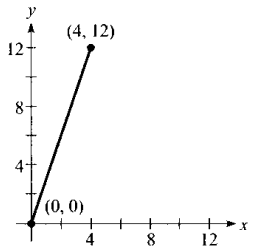
45. 4,83 millas por segundo

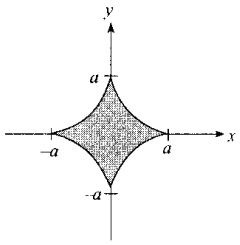
47. 4,67 millas por segundo

Sección 11.5 *(página 1095)*

1. $4\sqrt{10}$

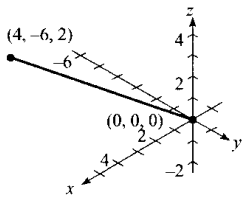
3. $6a$

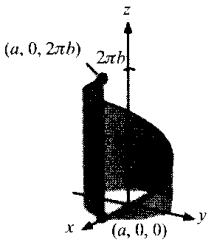




5. $2\sqrt{14}$

7. $2\pi\sqrt{a^2 + b^2}$





9. 8,37

11. *a*) $2\sqrt{21} \approx 9{,}165$

b) 9,529

c) Aumentar el número de segmentos.

d) 9,571

13. *a*) $s = \sqrt{5}t$

b) $\mathbf{r}(t) = 2\cos\frac{s}{\sqrt{5}}\mathbf{i} + 2\operatorname{sen}\frac{s}{\sqrt{5}}\mathbf{j} + \frac{s}{\sqrt{5}}\mathbf{k}$

c) $s = \sqrt{5}$: (1,081, 1,683, 1,000)
$s = 4$: (−0,433, 1,953, 1,789)

15. $K = 0$

17. $K = \frac{2}{5}$

19. $K = 0$

21. $\frac{\sqrt{2}}{2}$

23. $\frac{1}{4}$

25. $\frac{1}{a}$

27. $\frac{\sqrt{2}}{2}e^{-t}$

29. $\frac{1}{\omega t}$

31. $\frac{\sqrt{5}}{(1 + 5t^2)^{3/2}}$

33. $\frac{3}{25}$

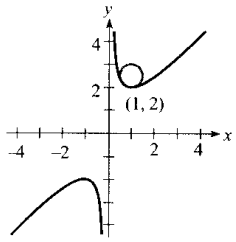
35. $K = 0$, $\frac{1}{K}$ no está definido

37. $K = \dfrac{4}{17^{3/2}}, \dfrac{1}{K} = \dfrac{17^{3/2}}{4}$ **39.** $K = \dfrac{1}{a}, \dfrac{1}{K} = a$

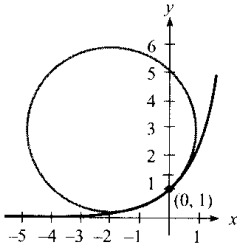
41. *a)* $\left(x - \dfrac{\pi}{2}\right)^2 + y^2 = 1$

b) Como la curvatura es menor, el radio de curvatura es mayor.

43. $(x - 1)^2 + \left(y - \dfrac{5}{2}\right)^2 = \left(\dfrac{1}{2}\right)^2$



45. $(x + 2)^2 + (y - 3)^2 = 8$



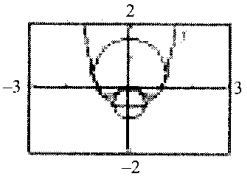
47. *a)* (1, 3) *b)* 0

49. *a)* $K \to \infty$ cuando $x \to 0$ *b)* 0 **51.** (1, 3)

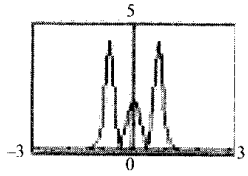
55. *a)* $K = \dfrac{2|6x^2 - 1|}{(16x^6 - 16x^4 + 4x^2 + 1)^{3/2}}$

b) $x = 0$: $x^2 + \left(y + \dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$x = 1$: $x^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{5}{4}$



c)



La curvatura tiende a ser mayor cerca de los extremos de la función y decrece cuando $x \to \pm\infty$. Sin embargo, f y K no tienen los mismos números críticos.

Números críticos de f: $x = 0, \pm\dfrac{\sqrt{2}}{2} \approx \pm 0{,}7071$

Números críticos de K: $x = 0, \pm 0{,}7647, \pm 0{,}4082$

57. *a)* 12,25 unidades *b)* $\frac{1}{2}$

59. En un extremo relativo suave $K = |y''|$. Sí. $y = x^4$ tiene curvatura 0 en su mínimo (0, 0). La curvatura es positiva en todos los demás puntos de la curva.

61. (−2, 3) **63.** $\dfrac{3}{2\sqrt{2(1 + \operatorname{sen}\theta)}}$

65. $\dfrac{2}{|a|}$ **67.** *a)* 0 *b)* 0

69. $\dfrac{1}{4}$ **73.** $K = \dfrac{1}{4a}\left|\operatorname{cosec}\dfrac{\theta}{2}\right|$

75. 2.420 libras Mínimo: $K = \dfrac{1}{4a}$

No hay máximo

85. *a)* a *b)* πa *c)* $K = \pi a$

Ejercicios de repaso del Capítulo 11 *(página 1099)*

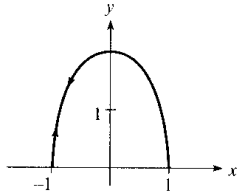
1. *a)* Todos los reales excepto $n\pi$, con n entero.
b) Continua excepto en $t = n\pi$, n entero.

3. *a)* $(0, \infty)$ *b)* Continua

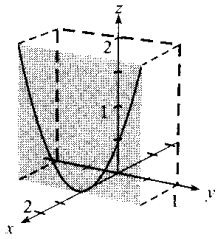
5. *a)* $\mathbf{i}$ *b)* $-3\mathbf{i} + 4\mathbf{j} + \frac{8}{3}\mathbf{k}$
c) $(2c - 1)\mathbf{i} + (c - 1)^2\mathbf{j} + \frac{1}{3}(1 - c)^3\mathbf{k}$
d) $2\Delta t\mathbf{i} + \Delta t(\Delta t + 2)\mathbf{j} - \frac{1}{3}\Delta t[(\Delta t)^2 + 3\Delta t + 3]\mathbf{k}$

7. $1 - t - \operatorname{sen} t$ No: el producto escalar es un número.

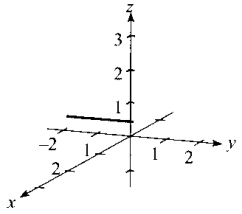
9.



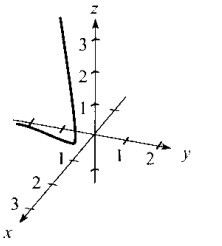
11.



13.



15.

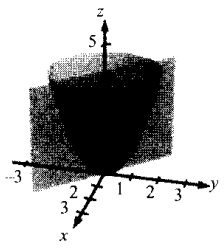


17. $\mathbf{r}_1(t) = 4t\mathbf{i} + 3t\mathbf{j}$, $0 \leqslant t \leqslant 1$
$\mathbf{r}_2(t) = 4\mathbf{i} + (3 - t)\mathbf{j}$, $0 \leqslant t \leqslant 3$
$\mathbf{r}_3(t) = (4 - t)$, $0 \leqslant t \leqslant 4$

19. $\mathbf{r}(t) = \langle -2 + 7t, -3 + 4t, 8 - 10t \rangle$

21. $x = t,\ y = -t,\ z = 2t^2$

23. $4\mathbf{i} + \mathbf{k}$



25. *a)* $3\mathbf{i} + \mathbf{j}$ *b)* $\mathbf{0}$ *c)* $4t + 3t^2$

d) $-5\mathbf{i} + (2t - 2)\mathbf{j} + 2t^2\mathbf{k}$ *e)* $\dfrac{10t - 1}{\sqrt{10t^2 - 2t + 1}}$

f) $\left(\dfrac{8}{3}t^3 - 2t^2\right)\mathbf{i} - 8t^3\mathbf{j} + (9t^2 - 2t + 1)\mathbf{k}$

27. $x = -\sqrt{2} - \sqrt{2}t$

$y = \sqrt{2} - \sqrt{2}t$

$z = \dfrac{3\pi}{4} + t$

29. En $t = t_0$, $x(t)$ e $y(t)$ son funciones crecientes y $z(t)$ es decreciente.

31. $x(t) = t,\ y(t) = 16 + 8t,\ z(t) = 2 + \frac{1}{2}t$

$\mathbf{r}(4,1) \approx (0{,}1,\ 16{,}8,\ 2{,}05)$

33. $\operatorname{sen} t\mathbf{i} + (t \operatorname{sen} t + \cos t)\mathbf{j} + \mathbf{C}$

35. $\frac{1}{2}\left(t\sqrt{1 + t^2} + \ln\left|t + \sqrt{1 + t^2}\right|\right) + \mathbf{C}$

37. $\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + (e^t + 2)\mathbf{j} - (e^{-t} + 4)\mathbf{k}$

39. $\frac{32}{3}\mathbf{j}$

41. $\mathbf{v}(t) = \langle -3\cos^2 t \operatorname{sen} t,\ 3 \operatorname{sen}^2 t \cos t,\ 3\rangle$

$\|\mathbf{v}(t)\| = 3\sqrt{\operatorname{sen}^2 t \cos^2 t + 1}$

$\mathbf{a}(t) = \langle 3\cos t(3 \operatorname{sen}^2 t - 1),\ 3 \operatorname{sen} t(2\cos^2 t - \operatorname{sen}^2 t),\ 0\rangle$

43. 152 pies

45. 34,9 m/s

47.
$\mathbf{v} = 5\mathbf{i}$
$\|\mathbf{v}\| = 5$
$\mathbf{a} = \mathbf{0}$
$\mathbf{a} \cdot \mathbf{T} = 0$
$\mathbf{a} \cdot \mathbf{N}$ no existe
$K = 0$

49.
$\mathbf{v} = \mathbf{i} + \dfrac{1}{2\sqrt{t}}\mathbf{j}$
$\|\mathbf{v}\| = \dfrac{\sqrt{4t + 1}}{2\sqrt{t}}$
$\mathbf{a} = -\dfrac{1}{4t\sqrt{t}}\mathbf{j}$
$\mathbf{a} \cdot \mathbf{T} = \dfrac{-1}{4t\sqrt{t}\sqrt{4t + 1}}$
$\mathbf{a} \cdot \mathbf{N} = \dfrac{1}{2t\sqrt{4t + 1}}$
$K = \dfrac{2}{(4t + 1)^{3/2}}$

51.
$\mathbf{v} = e^t\mathbf{i} - e^{-t}\mathbf{j}$
$\|\mathbf{v}\| = \sqrt{e^{2t} + e^{-2t}}$
$\mathbf{a} = e^t\mathbf{i} + e^{-t}\mathbf{j}$
$\mathbf{a} \cdot \mathbf{T} = \dfrac{e^{2t} - e^{-2t}}{\sqrt{e^{2t} + e^{-2t}}}$
$\mathbf{a} \cdot \mathbf{N} = \dfrac{2}{\sqrt{e^{2t} + e^{-2t}}}$
$K = \dfrac{2}{(e^{2t} + e^{-2t})^{3/2}}$

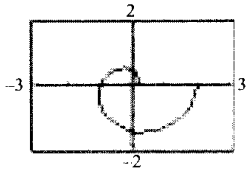
53.
$\mathbf{v} = \mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$
$\|\mathbf{v}\| = \sqrt{1 + 5t^2}$
$\mathbf{a} = 2\mathbf{j} + \mathbf{k}$
$\mathbf{a} \cdot \mathbf{T} = \dfrac{5t}{\sqrt{1 + 5t^2}}$
$\mathbf{a} \cdot \mathbf{N} = \dfrac{\sqrt{5}}{\sqrt{1 + 5t^2}}$
$K = \dfrac{\sqrt{5}}{(1 + 5t^2)^{3/2}}$

55. $\dfrac{\sqrt{5}\pi}{2}$

57. 4,56 millas por segundo

59. La curvatura cambia abruptamente de cero a un valor constante no nulo.

61. *a)*



b) $\sqrt{4\pi^2 + 1} + \dfrac{1}{2\pi}\ln\left(\sqrt{4\pi^2 + 1} + 2\pi\right) \approx 6{,}77$

c) $K = \dfrac{\pi(\pi^2 t^2 + 2)}{(\pi^2 t^2 + 1)^{3/2}}$

$K(0) = 2\pi,\ K(1) = \dfrac{\pi(\pi^2 + 2)}{(\pi^2 + 1)^{3/2}} \approx 1{,}04$

$K(2) = \dfrac{2\pi(2\pi^2 + 1)}{(4\pi^2 + 1)^{3/2}} \approx 0{,}51$

d)

e) 0

f) Conforme la gráfica describe la espiral hacia fuera, la curvatura va disminuyendo.

CAPÍTULO 12

Sección 12.1 *(página 1112)*

1. z es función de x e y

3. z no es función de x e y

5. *a)* $\dfrac{3}{2}$ *b)* $-\dfrac{1}{4}$ *c)* 6 *d)* $\dfrac{5}{y}$ *e)* $\dfrac{x}{2}$ *f)* $\dfrac{5}{t}$

7. *a)* 5 *b)* $3e^2$ *c)* $\dfrac{2}{e}$ *d)* $5e^y$ *e)* xe^2 *f)* te^t