7.4 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

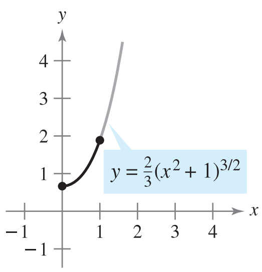
In Exercises 1 and 2, find the distance between the points using (a) the Distance Formula and (b) integration.

1. $(0, 0), \quad (8, 15)$

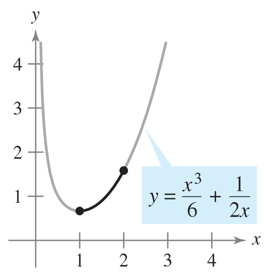
2. $(1, 2), \quad (7, 10)$

In Exercises 3–16, find the arc length of the graph of the function over the indicated interval.

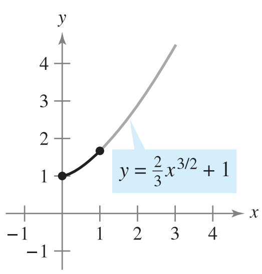
3. $y = \frac{2}{3}(x^2 + 1)^{3/2}$



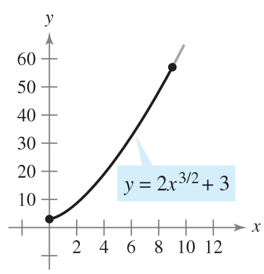
4. $y = \frac{x^3}{6} + \frac{1}{2x}$



5. $y = \frac{2}{3}x^{3/2} + 1$



6. $y = 2x^{3/2} + 3$



7. $y = \frac{3}{2}x^{2/3}, \quad [1, 8]$

8. $y = \frac{x^4}{8} + \frac{1}{4x^2}, \quad [1, 3]$

9. $y = \frac{x^5}{10} + \frac{1}{6x^3}, \quad [2, 5]$

10. $y = \frac{3}{2}x^{2/3} + 4, \quad [1, 27]$

11. $y = \ln(\sin x), \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

12. $y = \ln(\cos x), \quad \left[0, \frac{\pi}{3}\right]$

13. $y = \frac{1}{2}(e^x + e^{-x}), \quad [0, 2]$

14. $y = \ln\left(\frac{e^x + 1}{e^x - 1}\right), \quad [\ln 2, \ln 3]$

15. $x = \frac{1}{3}(y^2 + 2)^{3/2}, \quad 0 \le y \le 4$

16. $x = \frac{1}{3}\sqrt{y}(y - 3), \quad 1 \le y \le 4$

In Exercises 17–26, (a) sketch the graph of the function, highlighting the part indicated by the given interval, (b) find a definite integral that represents the arc length of the curve over the indicated interval and observe that the integral cannot be evaluated with the techniques studied so far, and (c) use the integration capabilities of a graphing utility to approximate the arc length.

17. $y = 4 - x^2, \quad 0 \le x \le 2$

18. $y = x^2 + x - 2, \quad -2 \le x \le 1$

19. $y = \frac{1}{x}, \quad 1 \le x \le 3$

20. $y = \frac{1}{x + 1}, \quad 0 \le x \le 1$

21. $y = \sin x, \quad 0 \le x \le \pi$

22. $y = \cos x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

23. $x = e^{-y}, \quad 0 \le y \le 2$

24. $y = \ln x, \quad 1 \le x \le 5$

25. $y = 2 \arctan x, \quad 0 \le x \le 1$

26. $x = \sqrt{36 - y^2}, \quad 0 \le y \le 3$

Approximation **In Exercises 27 and 28, determine which value best approximates the length of the arc represented by the integral. (Make your selection on the basis of a sketch of the arc and *not* by performing any calculations.)**

27. $\displaystyle\int_0^2 \sqrt{1 + \left[\frac{d}{dx}\left(\frac{5}{x^2 + 1}\right)\right]^2}\, dx$

(a) 25 (b) 5 (c) 2 (d) -4 (e) 3

28. $\displaystyle\int_0^{\pi/4} \sqrt{1 + \left[\frac{d}{dx}(\tan x)\right]^2}\, dx$

(a) 3 (b) -2 (c) 4 (d) $\frac{4\pi}{3}$ (e) 1

Approximation **In Exercises 29 and 30, approximate the arc length of the graph of the function over the interval [0, 4] in four ways. (a) Use the Distance Formula to find the distance between the endpoints of the arc. (b) Use the Distance Formula to find the lengths of the four line segments connecting the points on the arc when $x = 0$, $x = 1$, $x = 2$, $x = 3$, and $x = 4$. Find the sum of the four lengths. (c) Use Simpson's Rule with $n = 10$ to approximate the integral yielding the indicated arc length. (d) Use the integration capabilities of a graphing utility to approximate the integral yielding the indicated arc length.**

29. $f(x) = x^3$

30. $f(x) = (x^2 - 4)^2$

31. ***Length of a Catenary*** Electrical wires suspended between two towers form a catenary (see figure) modeled by the equation

$$y = 20 \cosh \frac{x}{20}, \quad -20 \le x \le 20$$

where x and y are measured in meters. The towers are 40 meters apart. Find the length of the suspended cable.