Arc Length **In Exercises 45–48, write an integral that represents the arc length of the curve on the given interval. Do not evaluate the integral.**

	Parametric Equations	*Interval*
45.	$x = 3t - t^2, \quad y = 2t^{3/2}$	$1 \le t \le 3$
46.	$x = \ln t, \quad y = 4t - 3$	$1 \le t \le 5$
47.	$x = e^t + 2, \quad y = 2t + 1$	$-2 \le t \le 2$
48.	$x = t + \sin t, \quad y = t - \cos t$	$0 \le t \le \pi$

Arc Length **In Exercises 49–56, find the arc length of the curve on the given interval.**

	Parametric Equations	*Interval*
49.	$x = 3t + 5, \quad y = 7 - 2t$	$-1 \le t \le 3$
50.	$x = t^2, \quad y = 2t$	$0 \le t \le 2$
51.	$x = 6t^2, \quad y = 2t^3$	$1 \le t \le 4$
52.	$x = t^2 + 1, \quad y = 4t^3 + 3$	$-1 \le t \le 0$
53.	$x = e^{-t}\cos t, \quad y = e^{-t}\sin t$	$0 \le t \le \frac{\pi}{2}$
54.	$x = \arcsin t, \quad y = \ln\sqrt{1 - t^2}$	$0 \le t \le \frac{1}{2}$
55.	$x = \sqrt{t}, \quad y = 3t - 1$	$0 \le t \le 1$
56.	$x = t, \quad y = \frac{t^5}{10} + \frac{1}{6t^3}$	$1 \le t \le 2$

Arc Length **In Exercises 57–60, find the arc length of the curve on the interval $[0, 2\pi]$.**

57. Hypocycloid perimeter: $x = a\cos^3\theta, y = a\sin^3\theta$

58. Circle circumference: $x = a\cos\theta, y = a\sin\theta$

59. Cycloid arch: $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$

60. Involute of a circle: $x = \cos\theta + \theta\sin\theta, y = \sin\theta - \theta\cos\theta$

61. ***Path of a Projectile*** The path of a projectile is modeled by the parametric equations

$$x = (90\cos 30°)t \quad \text{and} \quad y = (90\sin 30°)t - 16t^2$$

where x and y are measured in feet.

(a) Use a graphing utility to graph the path of the projectile.

(b) Use a graphing utility to approximate the range of the projectile.

(c) Use the integration capabilities of a graphing utility to approximate the arc length of the path. Compare this result with the range of the projectile.

62. ***Path of a Projectile*** If the projectile in Exercise 61 is launched at an angle θ with the horizontal, its parametric equations are

$$x = (90\cos\theta)t \quad \text{and} \quad y = (90\sin\theta)t - 16t^2.$$

Use a graphing utility to find the angle that maximizes the range of the projectile. What angle maximizes the arc length of the trajectory?

63. ***Folium of Descartes*** Consider the parametric equations

$$x = \frac{4t}{1 + t^3} \quad \text{and} \quad y = \frac{4t^2}{1 + t^3}.$$

(a) Use a graphing utility to graph the curve represented by the parametric equations.

(b) Use a graphing utility to find the points of horizontal tangency to the curve.

(c) Use the integration capabilities of a graphing utility to approximate the arc length of the closed loop. (*Hint:* Use symmetry and integrate over the interval $0 \le t \le 1$.)

64. ***Witch of Agnesi*** Consider the parametric equations

$$x = 4\cot\theta \quad \text{and} \quad y = 4\sin^2\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$$

(a) Use a graphing utility to graph the curve represented by the parametric equations.

(b) Use a graphing utility to find the points of horizontal tangency to the curve.

(c) Use the integration capabilities of a graphing utility to approximate the arc length over the interval $\pi/4 \le \theta \le \pi/2$.

65. ***Writing***

(a) Use a graphing utility to graph each set of parametric equations.

$x = t - \sin t$	$x = 2t - \sin(2t)$
$y = 1 - \cos t$	$y = 1 - \cos(2t)$
$0 \le t \le 2\pi$	$0 \le t \le \pi$

(b) Compare the graphs of the two sets of parametric equations in part (a). If the curve represents the motion of a particle and t is time, what can you infer about the average speeds of the particle on the paths represented by the two sets of parametric equations?

(c) Without graphing the curve, determine the time required for a particle to traverse the same path as in parts (a) and (b) if the path is modeled by

$$x = \tfrac{1}{2}t - \sin\left(\tfrac{1}{2}t\right) \quad \text{and} \quad y = 1 - \cos\left(\tfrac{1}{2}t\right).$$

66. ***Writing***

(a) Each set of parametric equations represents the motion of a particle. Use a graphing utility to graph each set.

First Particle	*Second Particle*
$x = 3\cos t$	$x = 4\sin t$
$y = 4\sin t$	$y = 3\cos t$
$0 \le t \le 2\pi$	$0 \le t \le 2\pi$

(b) Determine the number of points of intersection.

(c) Will the particles ever be at the same place at the same time? If so, identify the point(s).

(d) Explain what happens if the motion of the second particle is represented by

$$x = 2 + 3\sin t, \quad y = 2 - 4\cos t, \quad 0 \le t \le 2\pi.$$