Parametric Equations Interval **45.** $x = 3t - t^2$, $y = 2t^{3/2}$ $1 \le t \le 3$

728

46.
$$x = \ln t$$
, $y = 4t - 3$ $1 \le t \le$

47.
$$x = e^t + 2$$
, $y = 2t + 1$ $-2 \le t \le 2$

48.
$$x = t + \sin t$$
, $y = t - \cos t$ $0 \le t \le \pi$

Arc Length In Exercises 49-56, find the arc length of the curve on the given interval.

Parametric Equations	Interval
49. $x = 3t + 5$, $y = 7 - 2t$	$-1 \le t \le 3$
50. $x = t^2$, $y = 2t$	$0 \le t \le 2$

51.
$$x = 6t^2$$
, $y = 2t^3$ $1 \le t \le 4$ **52.** $x = t^2 + 1$, $y = 4t^3 + 3$ $-1 \le t \le 0$

53.
$$x = e^{-t} \cos t$$
, $y = e^{-t} \sin t$ $0 \le t \le \frac{\pi}{2}$

54.
$$x = \arcsin t$$
, $y = \ln \sqrt{1 - t^2}$ $0 \le t \le \frac{1}{2}$

55.
$$x = \sqrt{t}, \quad y = 3t - 1$$
 $0 \le t \le 1$

56.
$$x = t$$
, $y = \frac{t^5}{10} + \frac{1}{6t^3}$ $1 \le t \le 3$

Arc Length In Exercises 57–60, find the arc length of the curve on the interval $[0, 2\pi]$.

- **57.** Hypocycloid perimeter: $x = a \cos^3 \theta$, $y = a \sin^3 \theta$
- **58.** Circle circumference: $x = a \cos \theta$, $y = a \sin \theta$
- **59.** Cycloid arch: $x = a(\theta \sin \theta)$, $y = a(1 \cos \theta)$
- **60.** Involute of a circle: $x = \cos \theta + \theta \sin \theta$, $y = \sin \theta \theta \cos \theta$
- 61. Path of a Projectile The path of a projectile is modeled by the parametric equations

$$x = (90 \cos 30^{\circ})t$$
 and $y = (90 \sin 30^{\circ})t - 16t^{2}$

where x and y are measured in feet.

- (a) Use a graphing utility to graph the path of the projectile.
- (b) Use a graphing utility to approximate the range of the projectile.
- (c) Use the integration capabilities of a graphing utility to approximate the arc length of the path. Compare this result with the range of the projectile.
- 62. Path of a Projectile If the projectile in Exercise 61 is launched at an angle θ with the horizontal, its parametric equations are

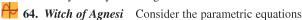
$$x = (90 \cos \theta)t$$
 and $y = (90 \sin \theta)t - 16t^2$.

Use a graphing utility to find the angle that maximizes the range of the projectile. What angle maximizes the arc length of the trajectory?

63. Folium of Descartes Consider the parametric equations

$$x = \frac{4t}{1+t^3}$$
 and $y = \frac{4t^2}{1+t^3}$.

- (a) Use a graphing utility to graph the curve represented by the parametric equations.
- (b) Use a graphing utility to find the points of horizontal tangency to the curve.
- (c) Use the integration capabilities of a graphing utility to approximate the arc length of the closed loop. (Hint: Use symmetry and integrate over the interval $0 \le t \le 1$.)



$$x = 4 \cot \theta$$
 and $y = 4 \sin^2 \theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

- (a) Use a graphing utility to graph the curve represented by the parametric equations.
- (b) Use a graphing utility to find the points of horizontal tangency to the curve.
- (c) Use the integration capabilities of a graphing utility to approximate the arc length over the interval $\pi/4 \le \theta \le \pi/2$.

65. Writing

(a) Use a graphing utility to graph each set of parametric equations.

$$x = t - \sin t \qquad x = 2t - \sin(2t)$$

$$y = 1 - \cos t \qquad y = 1 - \cos(2t)$$

$$0 \le t \le 2\pi \qquad 0 \le t \le \pi$$

- (b) Compare the graphs of the two sets of parametric equations in part (a). If the curve represents the motion of a particle and t is time, what can you infer about the average speeds of the particle on the paths represented by the two sets of parametric equations?
- (c) Without graphing the curve, determine the time required for a particle to traverse the same path as in parts (a) and (b) if the path is modeled by

$$x = \frac{1}{2}t - \sin(\frac{1}{2}t) \quad \text{and} \quad y = 1 - \cos(\frac{1}{2}t).$$

66. Writing

(a) Each set of parametric equations represents the motion of a particle. Use a graphing utility to graph each set.

First Particle	Second Particle
$x = 3 \cos t$	$x = 4 \sin t$
$y = 4 \sin t$	$y = 3 \cos t$
$0 \le t \le 2\pi$	$0 \le t \le 2\pi$

- (b) Determine the number of points of intersection.
- (c) Will the particles ever be at the same place at the same time? If so, identify the point(s).
- (d) Explain what happens if the motion of the second particle is represented by

$$x = 2 + 3\sin t$$
, $y = 2 - 4\cos t$, $0 \le t \le 2\pi$.